

# Comparison of Analytical and Empirical Performance Models

A Case Study on Multigrid Systems

Christian Kaltenecker  
University of Passau, Germany

Chair of  
Software  
Engineering



# Team



Christian  
Kaltenecker



Alexander  
Grebhahn



Sven  
Apel

# Configurable Systems

Which configuration is performance-optimal?



# Performance Models

- Mathematical formulas for estimating the performance

## Analytical

- Created by domain experts



- + Applicable on different hardware
- Requires domain knowledge
- Difficult for complex software

## Empirical

- Created by tools with measurement results of a specific hardware

**SPL**  
Conqueror

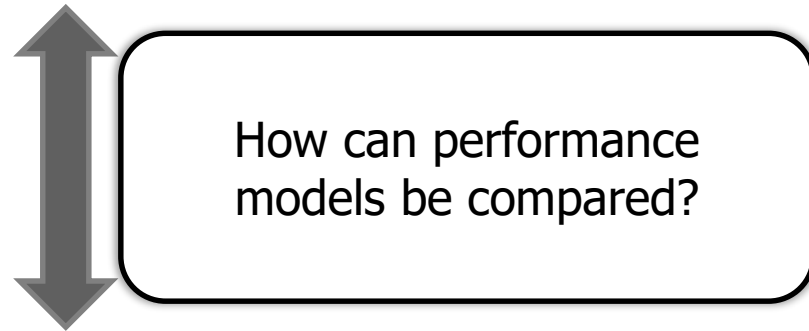


- + Applicable on complex systems
- ± Requires performance measurements
- Only applicable on measured hardware

# Performance Models

- Analytical:

$$23.37 * \log_2(px) + 23.37 * \log_2(nx) + 2.34 * 10^{-4} * nx$$



- Empirical:

$$-5 - 3 * px + 1.5 * px^2 + 10^{-5} * nx$$

# Comparison Strategies

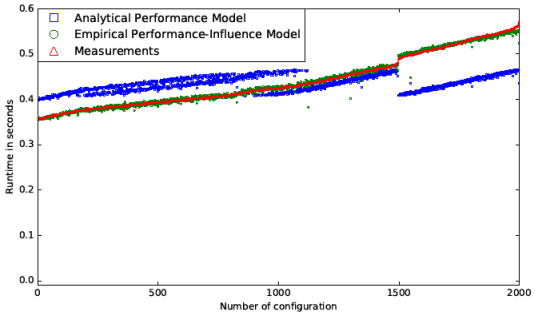
## Syntactic

Compares the coefficients of the variables

|                  | [ <i>nx</i> ]         |
|------------------|-----------------------|
| $T_{analytical}$ | $2.34 * 10^{-4} * nx$ |
| $T_{empirical}$  | $10^{-5} * nx$        |

## Semantic

Compares the prediction results



- Application of:
- Error Rate
  - Jaccard Similarity
  - Pearson Correlation

## Hybrid

Uses both syntactic and semantic elements for the comparison

|                  | [ <i>nx</i> ]         |
|------------------|-----------------------|
| $T_{analytical}$ | $2.34 * 10^{-4} * nx$ |
| $T_{empirical}$  | $10^{-5} * nx$        |

Calculate influence and use it as weight for Pearson correlation

# Conversion of the Performance Models

- Sort each term into equivalence classes

$$23.37 * \log_2(px) + 23.37 * \log_2(nx) + 2.34 * 10^{-4} * nx$$

|                               | [ <i>px</i> ]        | [ <i>nx</i> ]                              |
|-------------------------------|----------------------|--|
| <i>T<sub>analytical</sub></i> | $23.37 * \log_2(px)$ | $23.37 * \log_2(nx) ; 2.34 * 10^{-4} * nx$ |

# Syntactic Comparison

- Use the equivalence classes and compute score according to the formula:

$$\text{scoreOfTerms}(e, a) = \begin{cases} -1, & \text{if the other model has no such equivalence class} \\ 0, & \text{if the equivalence class exists, but no such term} \\ 1 + \text{simValue}(e, a), & \text{if the term exists in the other model} \end{cases}$$

$$\text{simValue}(e, a) = \max(0, 1 - \frac{|e-a|}{\max(e,a)})$$

$$23.37 * \log_2(px) + 23.37 * \log_2(nx) + 2.34 * 10^{-4} * nx$$

$$-5 - 3 * px + 10^{-5} * nx + 1.5 * px^2$$

|                  | [constant] | [px]                  | [nx]                                      |
|------------------|------------|-----------------------|---|
| $T_{analytical}$ |            | $23.37 * \log_2(px)$  | $23.37 * \log_2(nx); 2.34 * 10^{-4} * nx$ |
| $T_{empirical}$  | $-5$       | $-3 * px; 1.5 * px^2$ | $10^{-5} * nx$                            |
| Score            | -1         | 0                     | 1 + 0.43                                  |

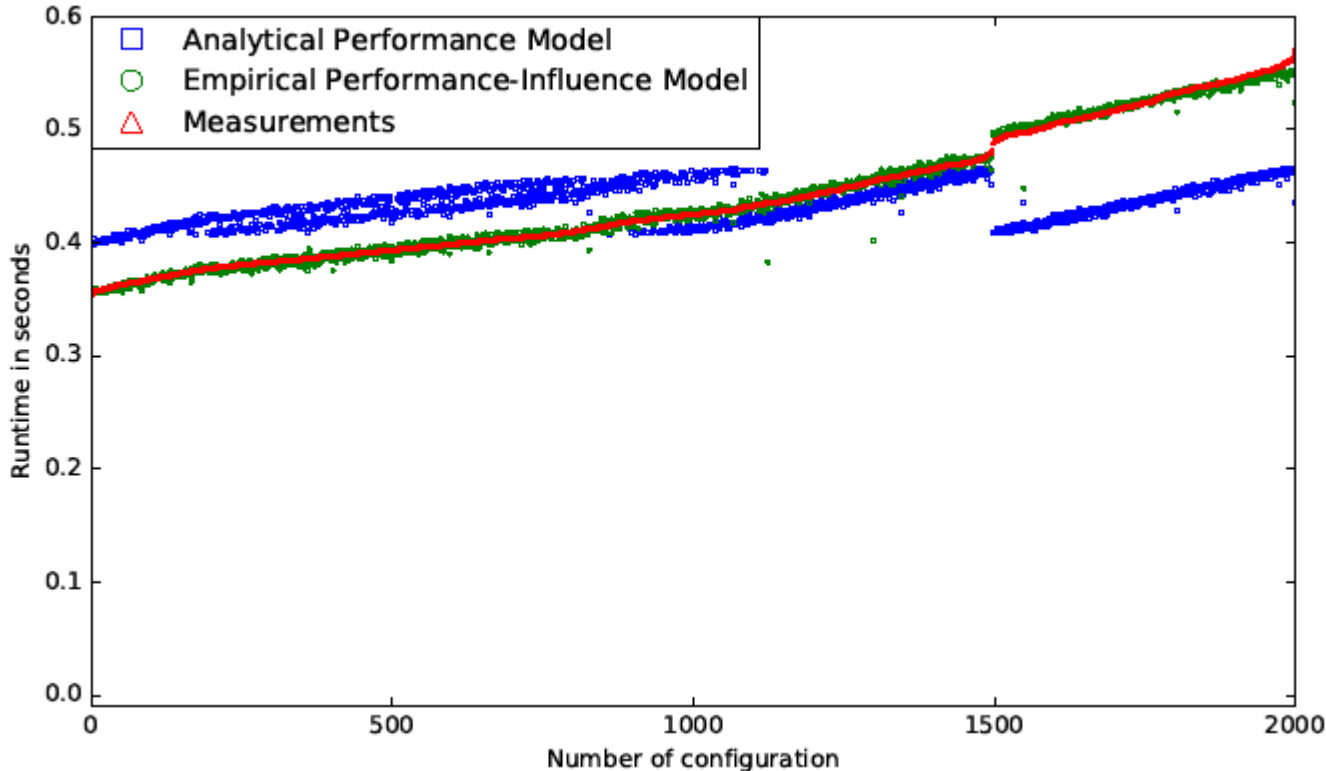


$$\text{Score} = 0.43$$



# Semantic Comparison

- Compute the results of the performance models



# Semantic Comparison

- Apply different measures on the computed results

| Name                | Result        |
|---------------------|---------------|
| Error Rate          | 10.1%         |
| Jaccard Similarity  | 90%           |
| Manhattan Distance  | 0.046         |
| Euclidean Distance  | 0.0029        |
| Pearson Correlation | 0.41 (medium) |

Good or bad?

# Hybrid Comparison

- Calculate the influence of the equivalence classes:

|                          | [ <i>constant</i> ] | [ <i>px</i> ]         | [ <i>nx</i> ]                             |
|--------------------------|---------------------|-----------------------|---|
| $T_{analytical}$         |                     | $23.37 * \log_2(px)$  | $23.37 * \log_2(nx); 2.34 * 10^{-4} * nx$ |
| $Influence_{analytical}$ | 0%                  | 1.1%                  | 98.9%                                     |
| $T_{empirical}$          | -5                  | $-3 * px; 1.5 * px^2$ | $10^{-5} * nx$                            |
| $Influence_{empirical}$  | 0%                  | 71.2%                 | 28.8%                                     |

- Use the similarity of the influences as a weight for the Pearson correlation between the equivalence classes  
Result: 0.3

# Conclusion

## Performance Models

- Mathematical formulas for estimating the performance

### Analytical

- Created by domain experts



- Applicable on different hardware

- Requires domain knowledge
- Difficult for complex software

### Empirical

- Created by tools with measurement results of a specific hardware



- Applicable on complex systems

- Requires performance measurements

- Only applicable on measured hardware

Comparison of Analytical and Empirical Performance Models

## Syntactic Comparison

- Use the equivalence classes and compute score according to the formula:

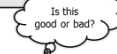
$$\text{scoreOfTerms}(e, a) = \begin{cases} 1 & \text{if the other model has no such equivalence class} \\ 0 & \text{if the equivalence class exists, but no such term} \\ \frac{1 + \text{simValue}(e, a)}{2} & \text{if the term exists in the other model} \end{cases}$$

$$\text{simValue}(e, a) = \max(0, 1 - \frac{|e - a|}{a})$$

$$23.37 \cdot \log_2(px) + 23.37 \cdot \log_2(nx) + 2.34 \cdot 10^{-4} \cdot nx$$

$$-5 - 3 \cdot px + 10^{-5} \cdot nx + 1.5 \cdot px^2$$

|                  | [constant] | [px]                          | [nx]  |
|------------------|------------|-------------------------------|---|
| $T_{analytical}$ |            | $23.37 \cdot \log_2(px)$      | $23.37 \cdot \log_2(nx); 2.34 \cdot 10^{-4} \cdot nx$ |
| $T_{empirical}$  | -5         | $-3 \cdot px; 1.5 \cdot px^2$ | $10^{-5} \cdot nx$                                    |
| Score            | -1         | 0                             | 1 + 0.43  |

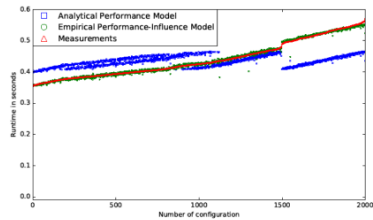


$$\text{Score} = 0.43$$

Comparison of Analytical and Empirical Performance Models

## Semantic Comparison

- Compute the results of the performance models



Comparison of Analytical and Empirical Performance Models

## Hybrid Comparison

- Calculate the influence of the equivalence classes:

|                          | [constant] | [px]                          | [nx]  |
|--------------------------|------------|-------------------------------|---|
| $T_{analytical}$         |            | $23.37 \cdot \log_2(px)$      | $23.37 \cdot \log_2(nx); 2.34 \cdot 10^{-4} \cdot nx$ |
| $Influence_{analytical}$ | 0%         | 27.7%                         | 72.3%   |
| $T_{empirical}$          | -5         | $-3 \cdot px; 1.5 \cdot px^2$ | $10^{-5} \cdot nx$                                    |
| $Influence_{empirical}$  | 0%         | 73%                           | 27%   |

- Use the influence as a weight for the Pearson correlation between the equivalence classes  
Result: 0.15 (weak)

Comparison of Analytical and Empirical Performance Models

**Thank you for your attention!**